Comparison of FK and SPAC Methods in Determining Dispersion Curves From Passive Surface Waves

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Outline

- Passive Surface Waves
- Dispersion Analysis Methods
- Comparison of Simulated Data
- Comparison of Field Data Examples
- Conclusion

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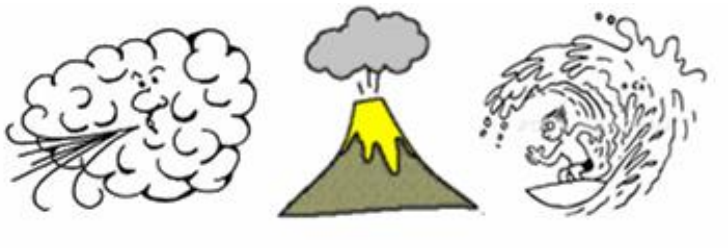
Passive Surface Waves – Sources

Passive waves are generated by:

1. Cultural sources – vehicle traffic, machinery, railways, etc.
2. Natural sources – wind noise, ocean wave, etc.



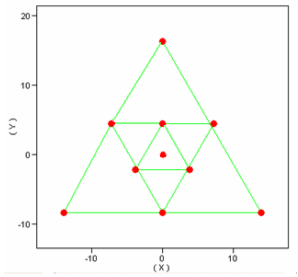
Different Terms:

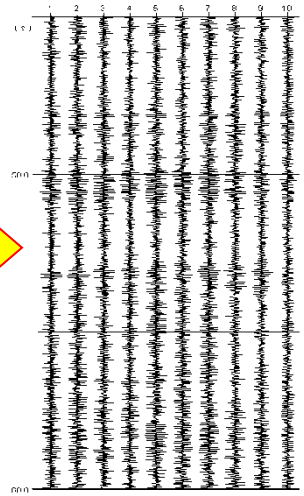
1. Passive Surface Wave (North America)
2. Microtremor (Japan)
3. Ambient Vibration (Europe)

Passive Surface Waves – Features

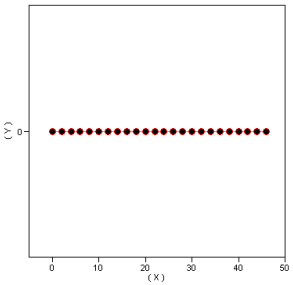
1. The sources are randomly distributed and triggered
2. The direction of wave propagation relative to the array are unknown
3. The frequency is lower compared with the active sources
4. Energy from such sources propagates principally as surface waves, and
5. **The dispersion curve can be extracted from such surface waves**

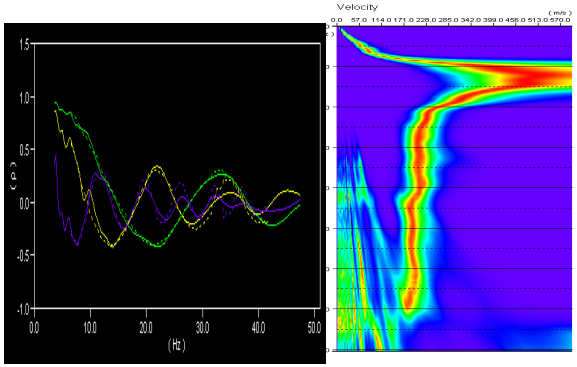
Passive Surface Waves – Procedure

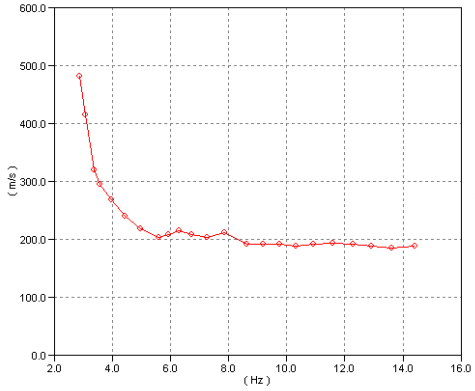


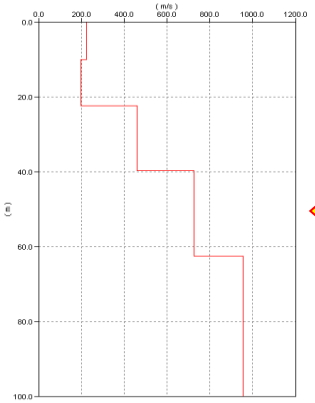


or









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Passive Surface Waves – Array

- 2D Array (Triangle; Circle; Cross; L shape)

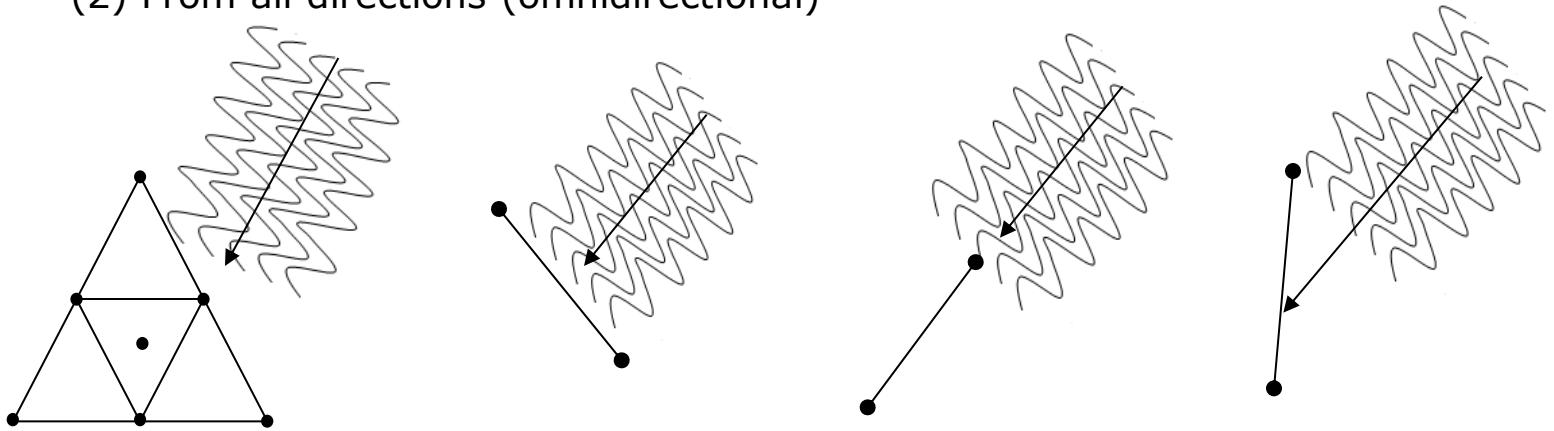
  Allow waves coming from any direction

- 1D Array (Linear)

  Assume waves are coming

  (1) Along the direction of array (roadside) or

  (2) From all directions (omnidirectional)



$V_m = V_R$ $V_m = \infty$ $V_m = V_R$ $V_R < V_m < \infty$

V_m: measured velocity; V_R: the true phase velocity

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Dispersion Analysis Methods

There are two methods to measure the phase velocity of passive surface waves:

1. SPAC / ESPAC

   Fitting the spatial autocorrelation coefficient

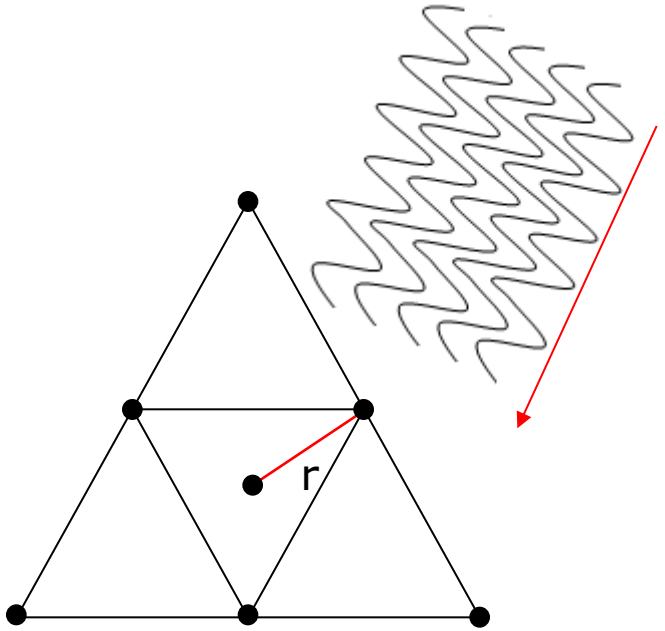
2. FK

   Azimuth scanning in FK, FV or FP domain

SPAC – Coherency

Provided a set of geophones azimuthally deployed around the circle, the coherency for a pair of geophones is:

$$C_{jc}(f) = \exp[ir_{jc}k\cos(\theta_{jc} - \phi)]$$



Where r_{jc} is the displacement of the jth geophone relative to the center at azimuthal angle θ_{jc}, k is the spatial wavenumber at frequency f, and Φ is the azimuth of propagation of a single plane wave across the array.

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SPAC – Azimuthal Average Coherency

If the number of pairs of geophones approaches to infinite, the spatial autocorrelation coefficient (azimuthal average of the coherency) can be expressed as:

$$c(f) = \frac{1}{2\pi}\int_0^{2\pi} \exp[irk\cos(\theta-\phi)]d\theta = J_0(rk) = J_0\left(\frac{2\pi fr}{v(f)}\right)$$

where J_0 is the Bessel function of the first kind of zero order.

(1) The spatial autocorrelation coefficient is a function of the phase velocity and frequency.

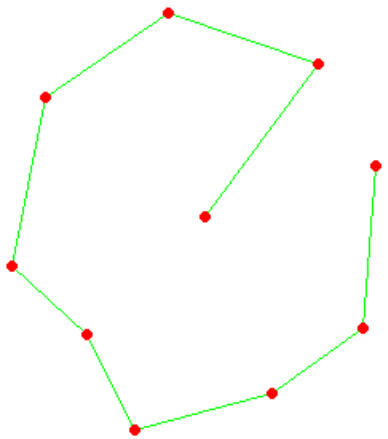
(2) The phase velocity $v(f)$ can be derived by fitting the spatial autocorrelation coefficient $c(f)$, which is calculated from the recorded signals, to the Bessel functions.

Note that this equation also demonstrates that the linear array is allowed if the passive sources are omnidirectional, i.e., integrated around the circle over φ with θ fixed.

ESPAC – Extended SPAC

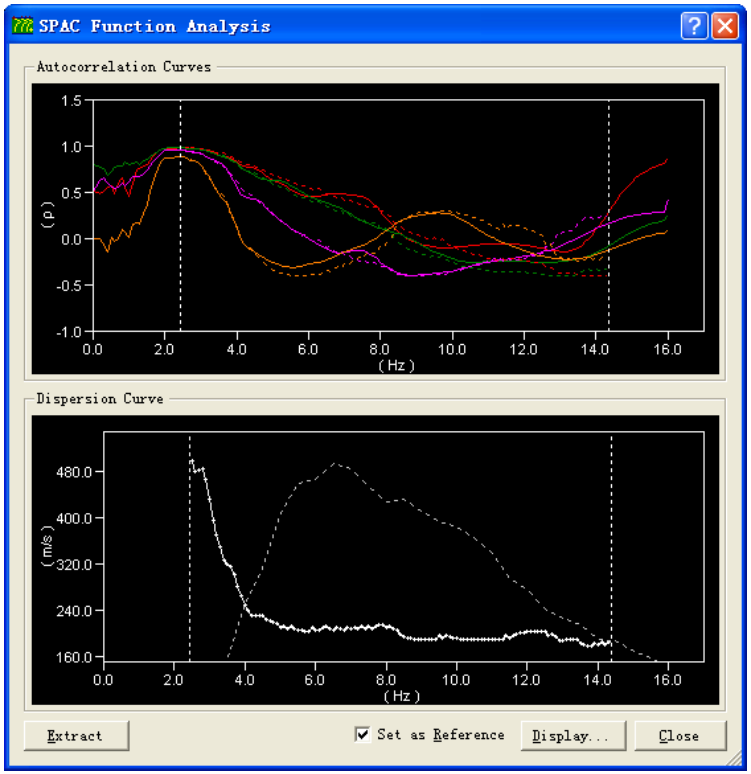
Practically it is sometimes difficult to deploy a regular array, such as circle and triangle.

SPAC has been improved to ESPAC (Extended Spatial Autocorrelation), which allows the configuration of irregular array.

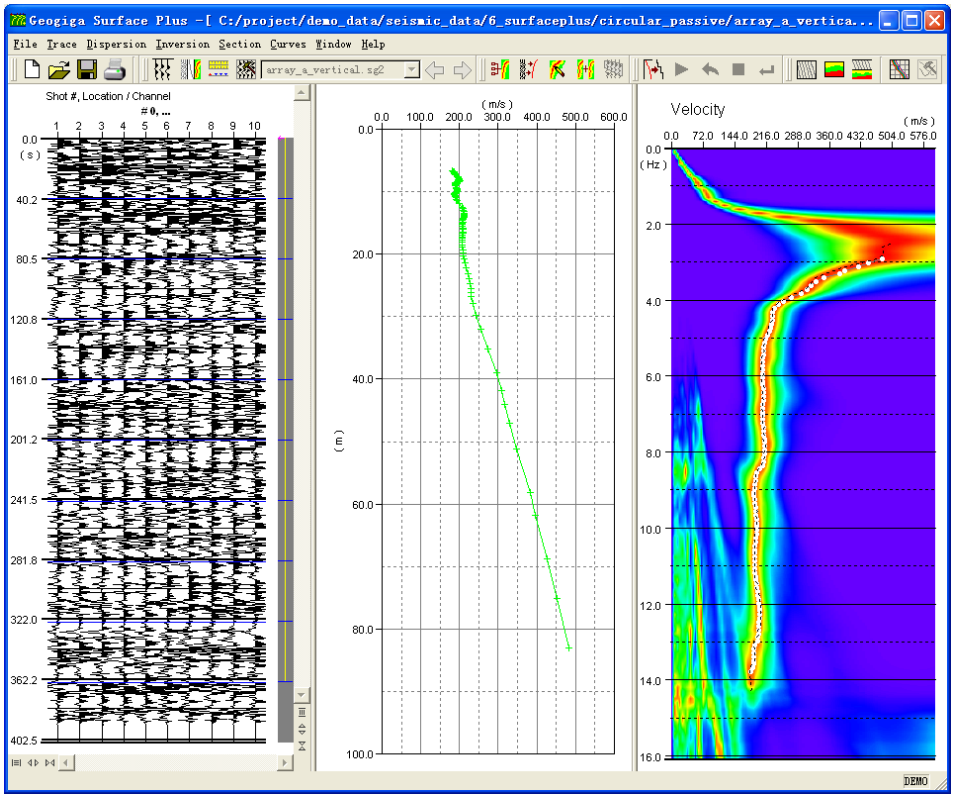


SPAC / ESPAC – Fitting and Imaging

Fitting spatial autocorrelation coefficients
to Bessel functions



Dispersion image of SPAC/ESPAC



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FK Method

FK method, computing a 2D frequency-wavenumber power spectrum from an array, has several forms. Here is the maximum likelihood:

$$P(k_x, k_y, f) = \left\{ \sum_{i=1}^{N} \sum_{j=1}^{N} \phi_{ij}(f) \exp\left[ik_x(x_i - x_j) + ik_y(y_i - y_j) \right] \right\}^{-1}$$

Where*: f is* frequency, (x_i , y_i) and (x_j , y_j) are the coordinates of receivers i and j, (k_x, k_y) is the wave number vector, $\phi_{ij}(f)$ is the correlation of signals at receivers i and j.

For a given frequency f_0, the location $k_0 = (k_{x0}, k_{y0})$ of the maximum semblance in the plane provides an estimate of velocity and azimuth of the wave traveling across the array, i.e.,

$$V_0 = \frac{2\pi f_0}{k_0} = \frac{2\pi f_0}{\sqrt{(k_{x0}^2 + k_{y0}^2)}} \quad , \qquad \theta = \tan^{-1}\left(\frac{k_{x0}}{k_{y0}} \right)$$

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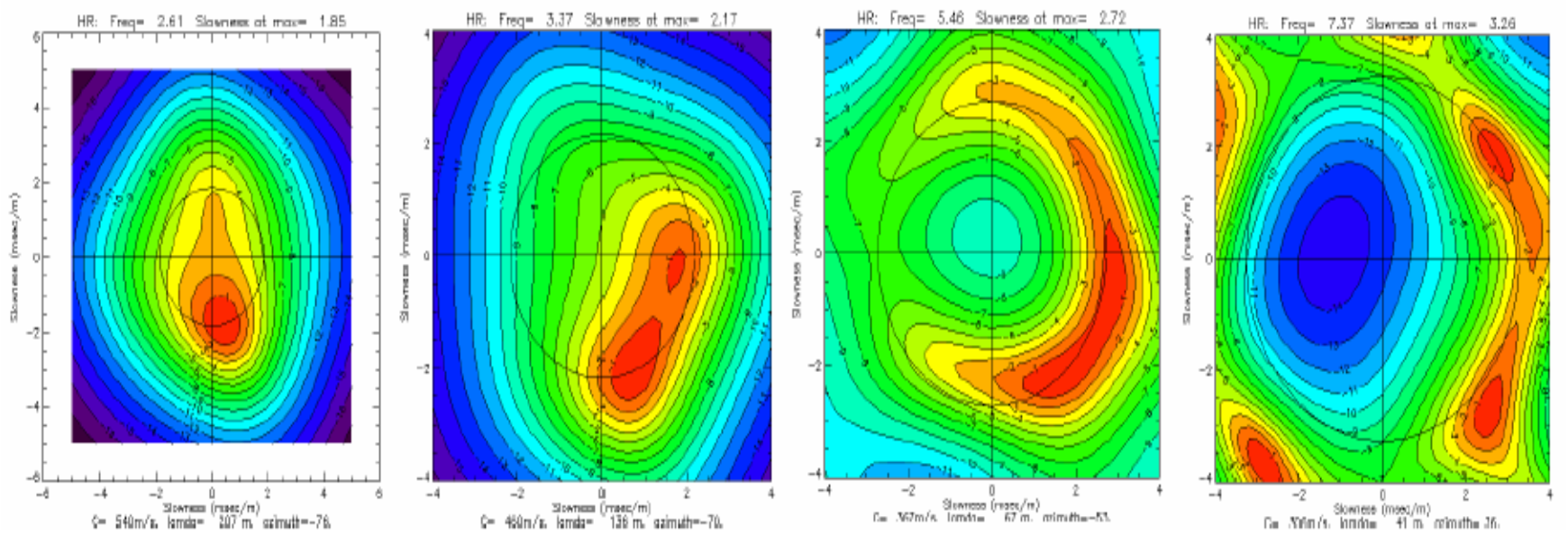
FK – Semblance Images

For a given frequency, scan all directions with 2d wavenumber



[Claprood, 2008]

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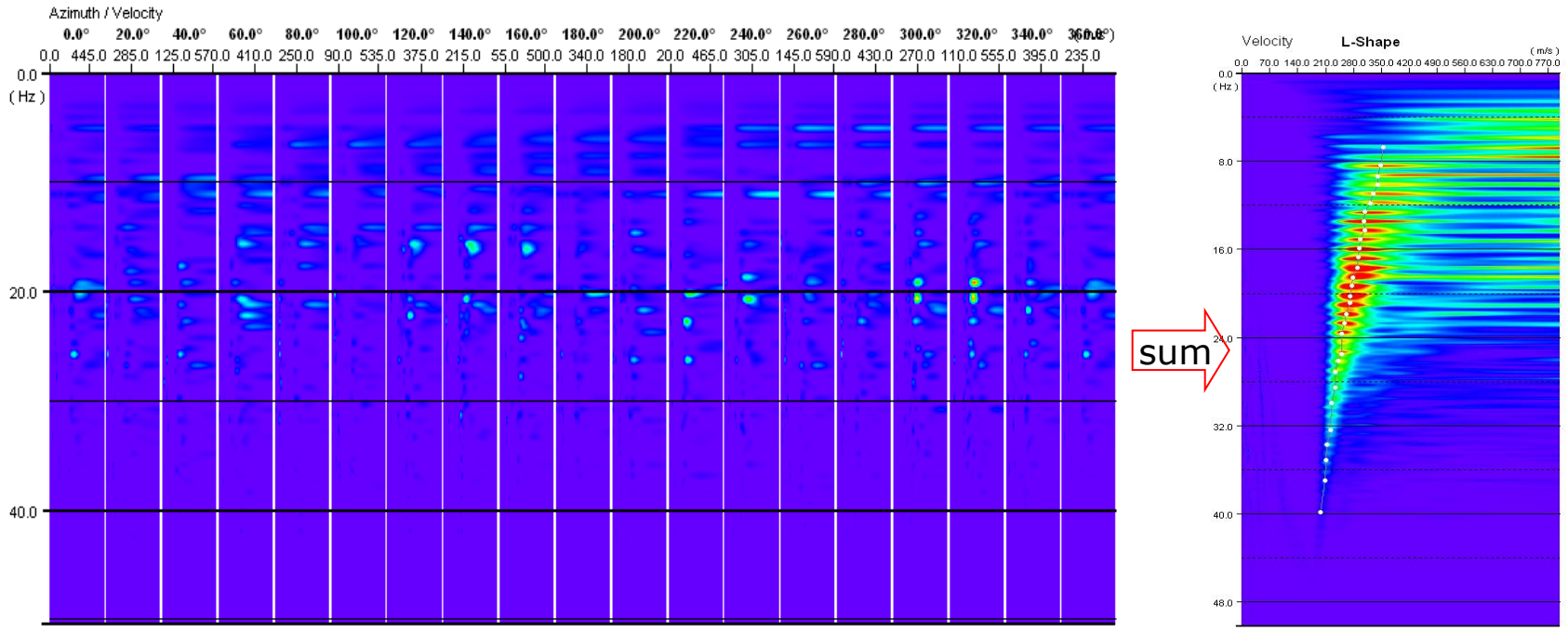
FK – Azimuth Scan

FK transform is conducted for all directions through the geophone array (azimuth scan), and the final dispersion image is the summation of all of them.



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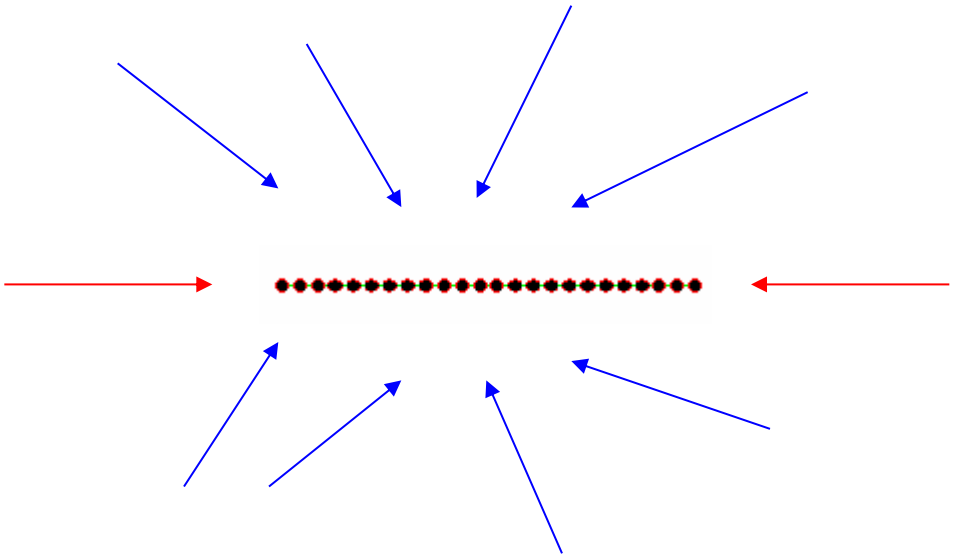
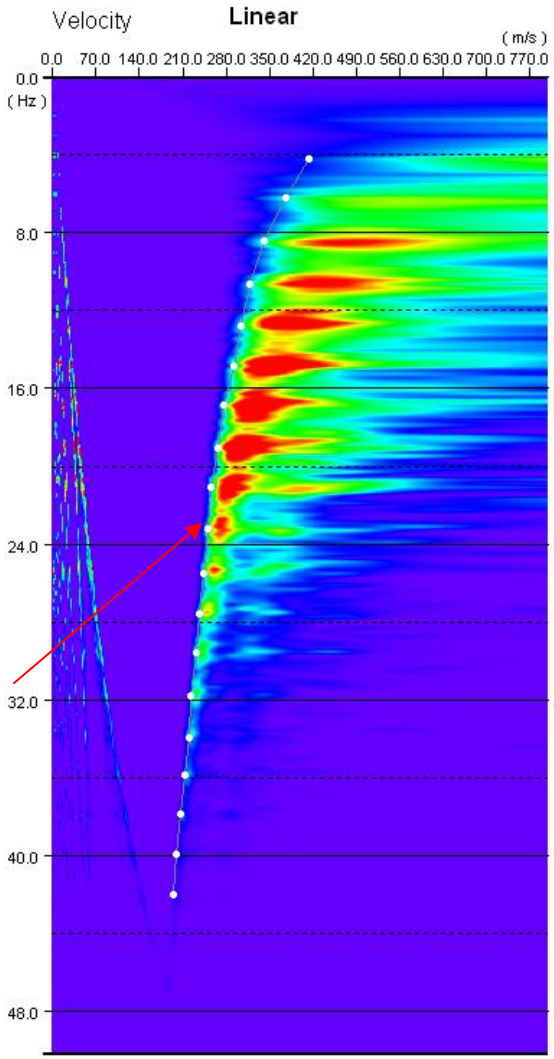
FK – Dispersion Picks for Linear Array

For the linear array, the dispersion curve of the passive surface wave should be picked along the envelope of the dispersion image with the slowest velocity (?).





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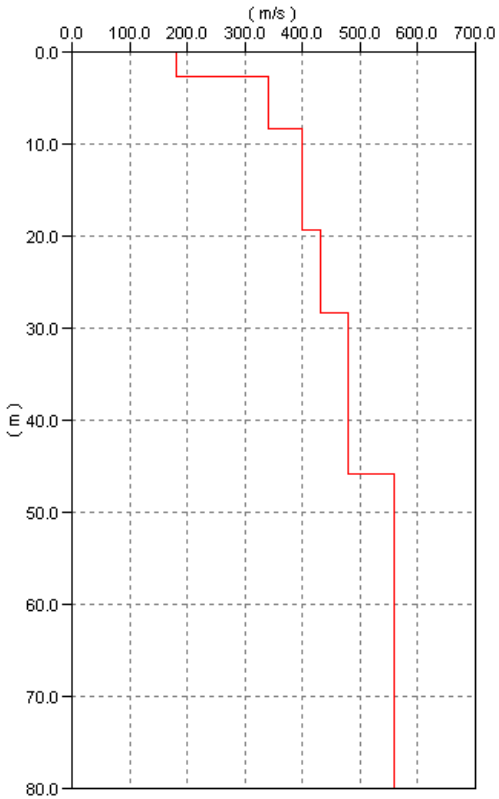
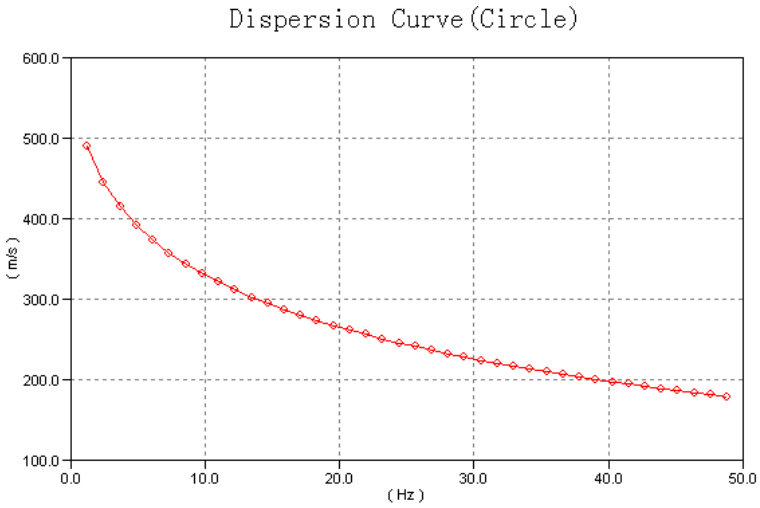
Passive Surface Waves – Simulation (Model)



Model



Dispersion Curve

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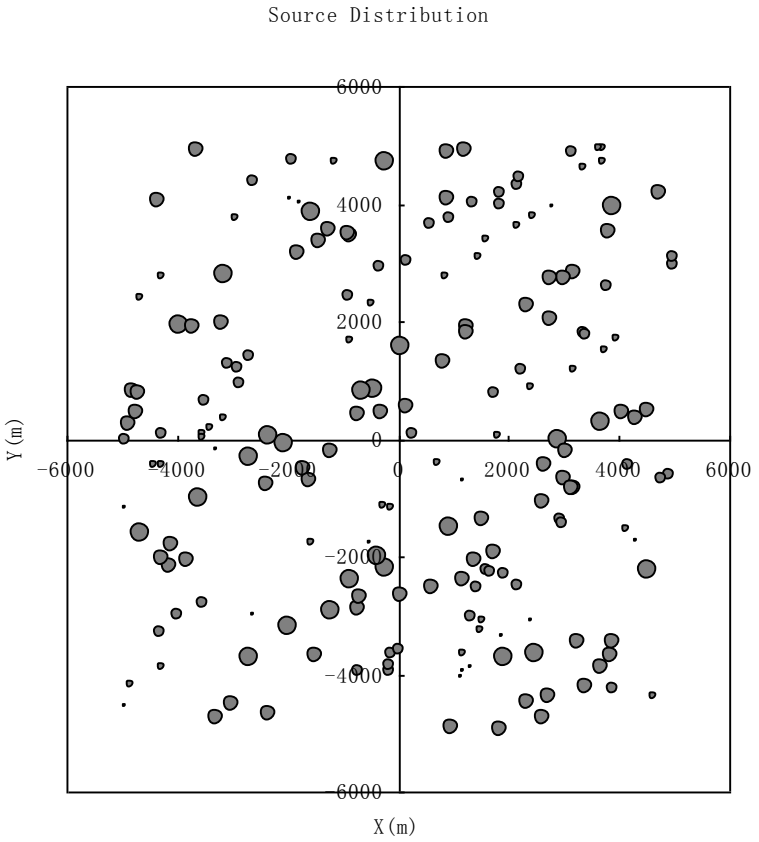
Passive Surface Waves – Simulation (Sources)



200 sources are randomly distributed with different strengths

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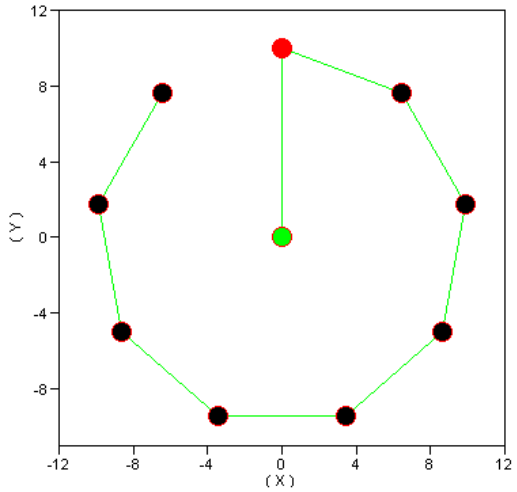
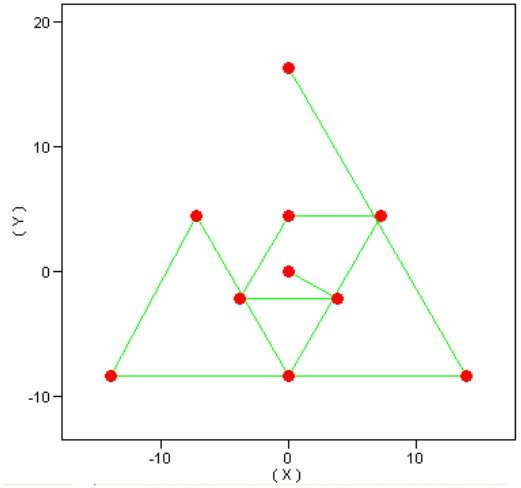
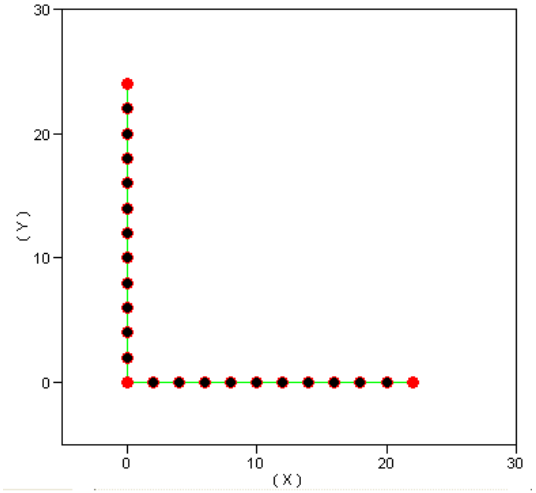
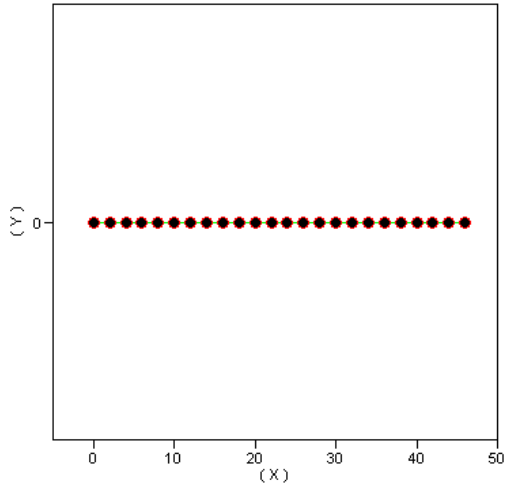
Passive Surface Waves – Simulation (Arrays)









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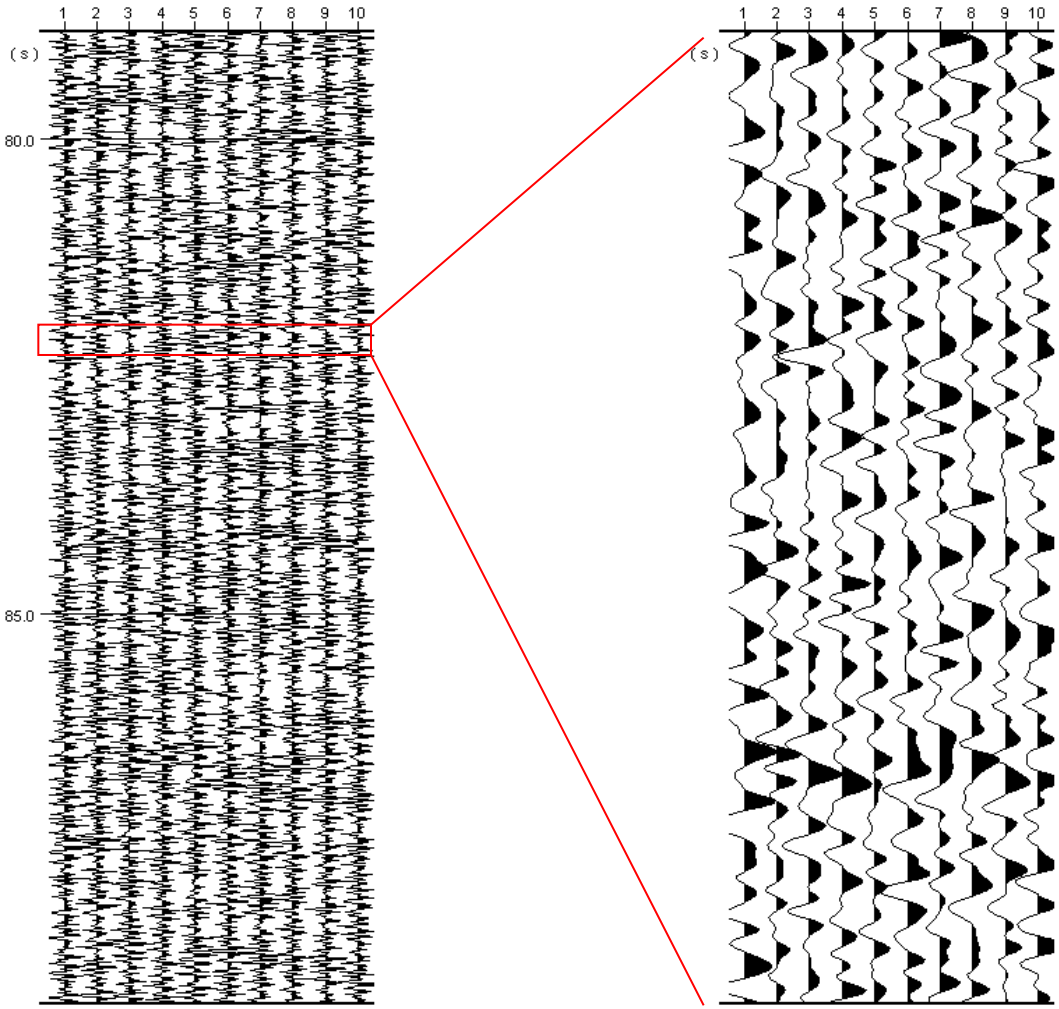
Passive Surface Waves – Simulation (Records)



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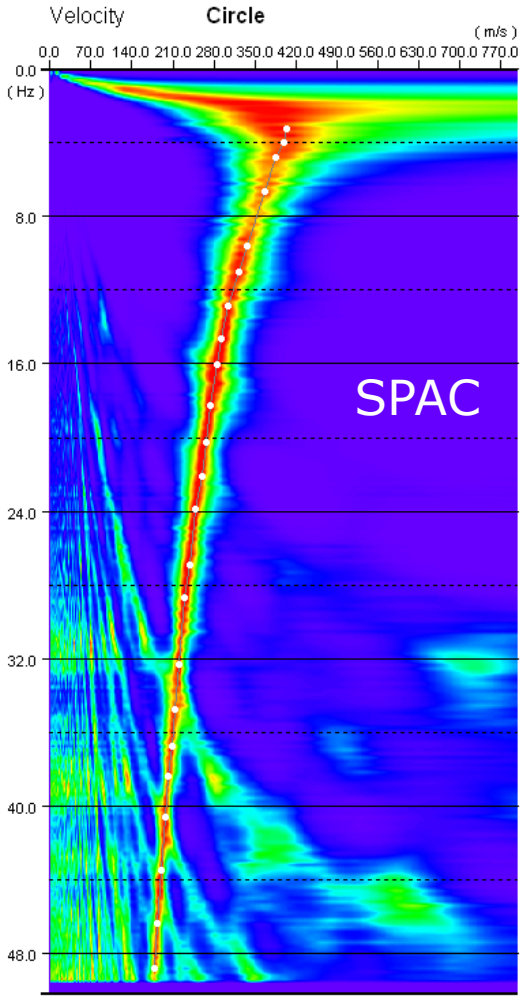
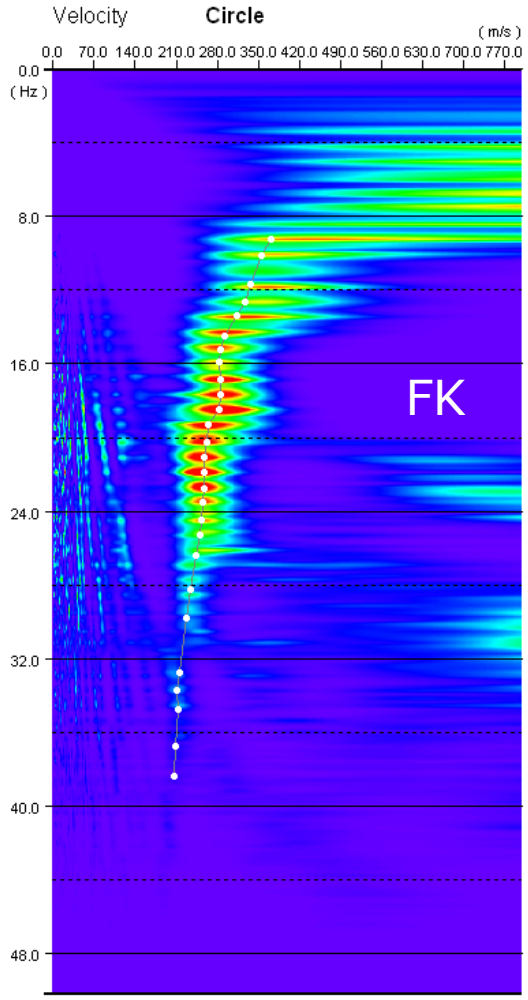
Dispersion Image Comparison - Circle





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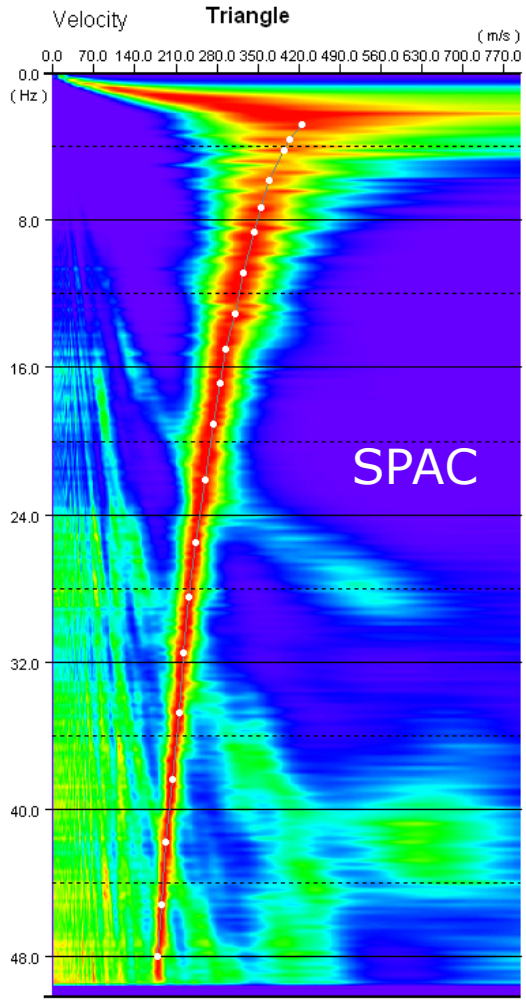
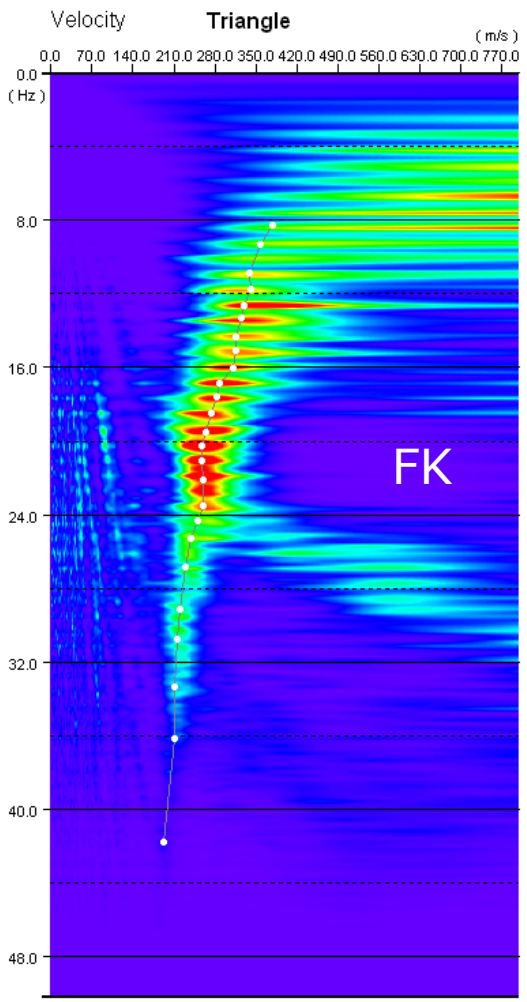
Dispersion Image Comparison – Triangle





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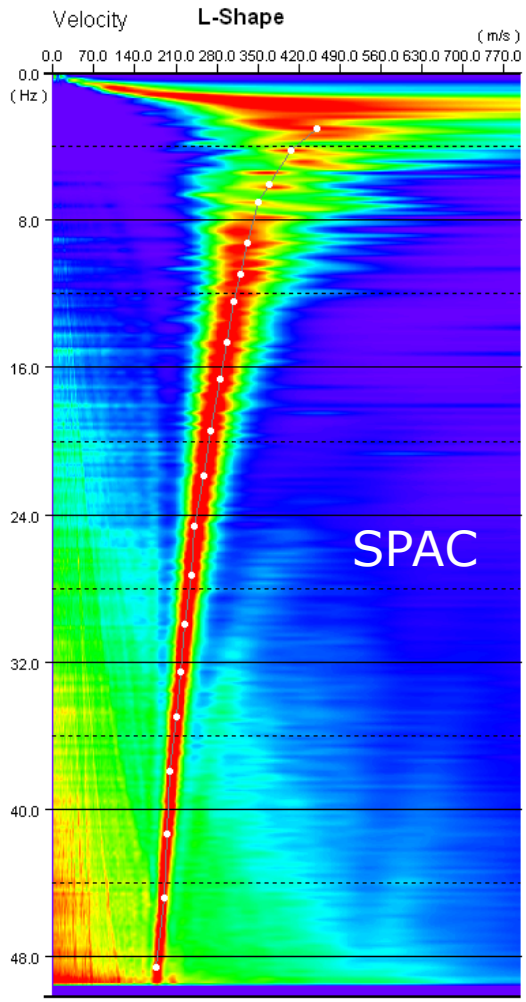
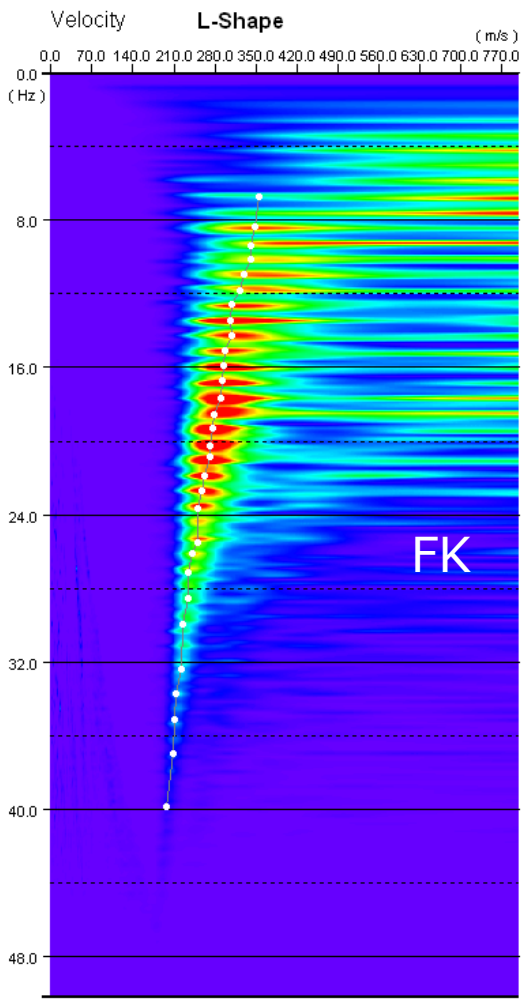
Dispersion Image Comparison – L-Shape





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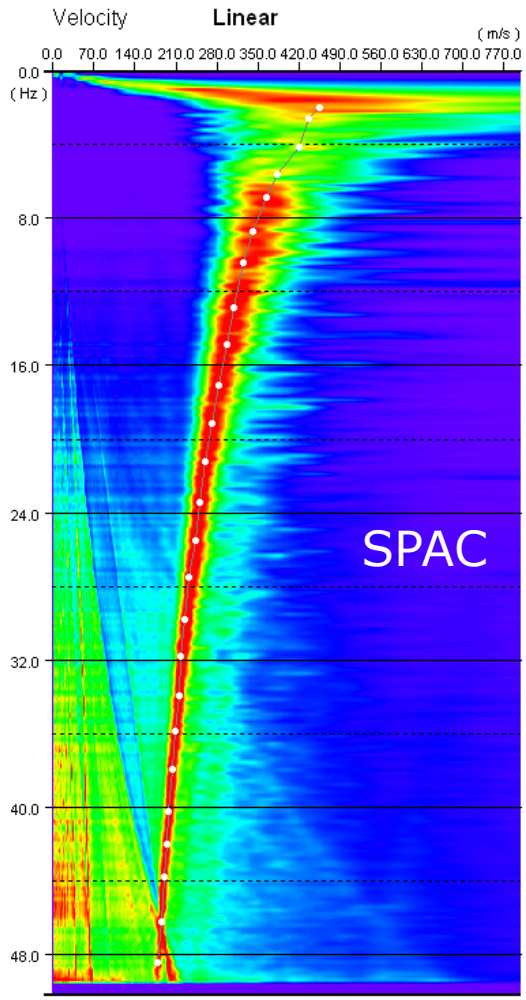
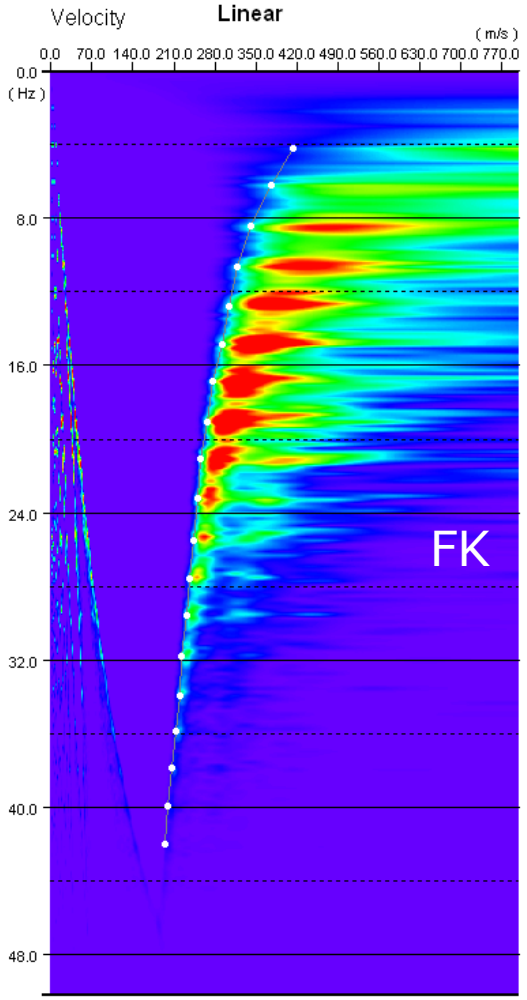
Dispersion Image Comparison – Linear





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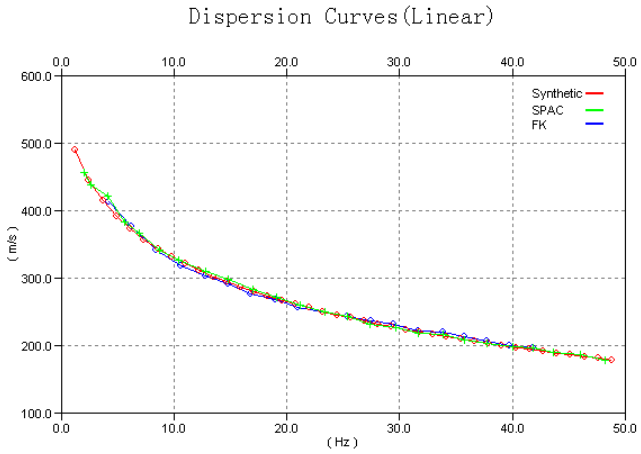
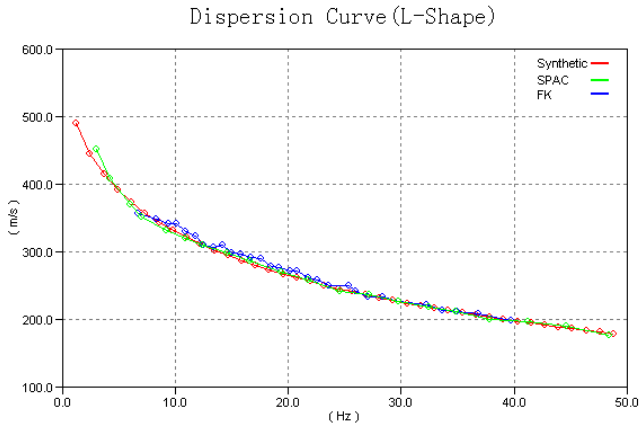
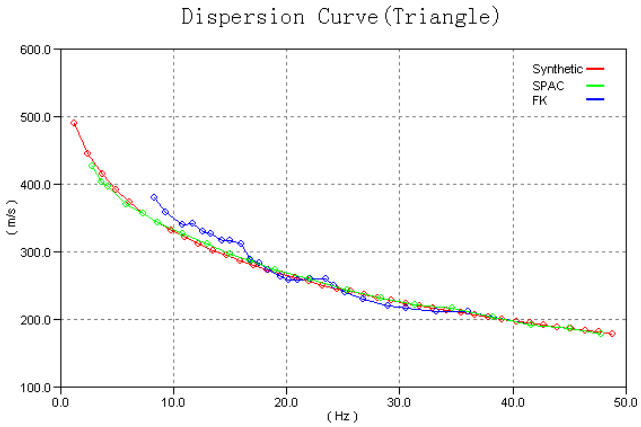
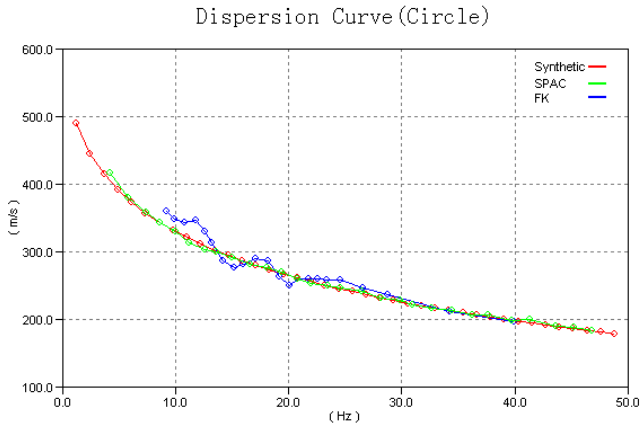
Simulation – Dispersion Curves









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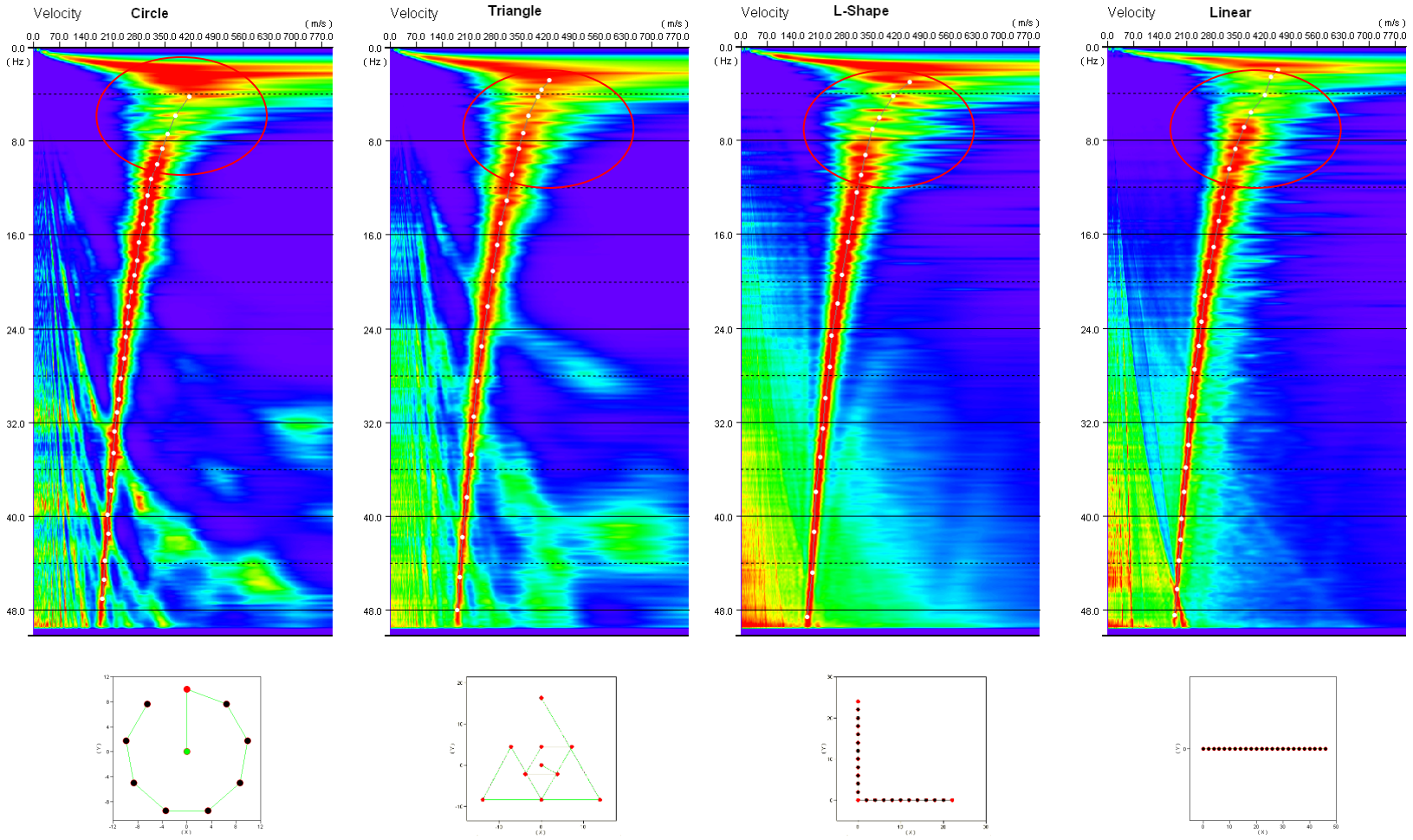
Simulation – Array Comparison (SPAC)



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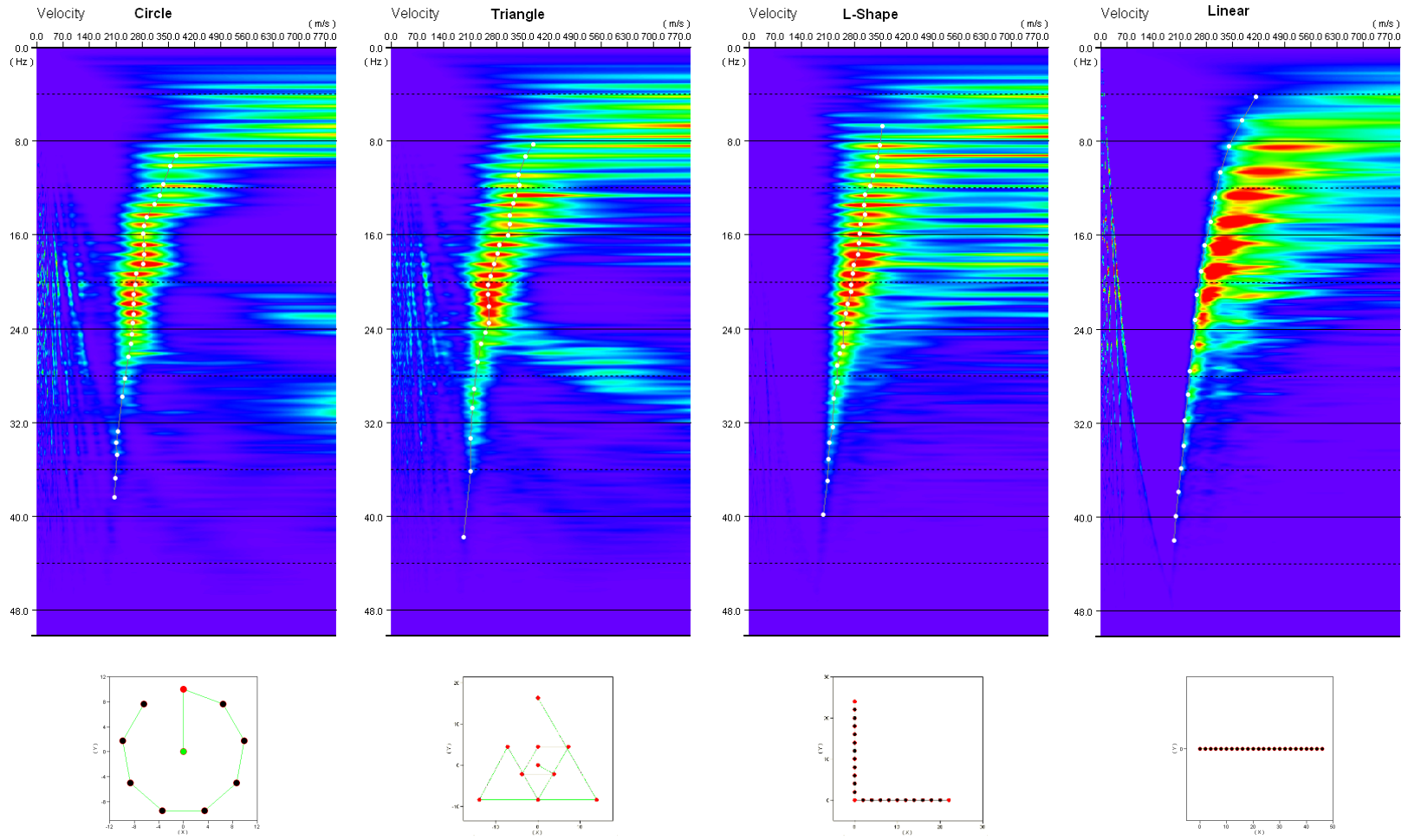
Simulation – Array Comparison (FK)



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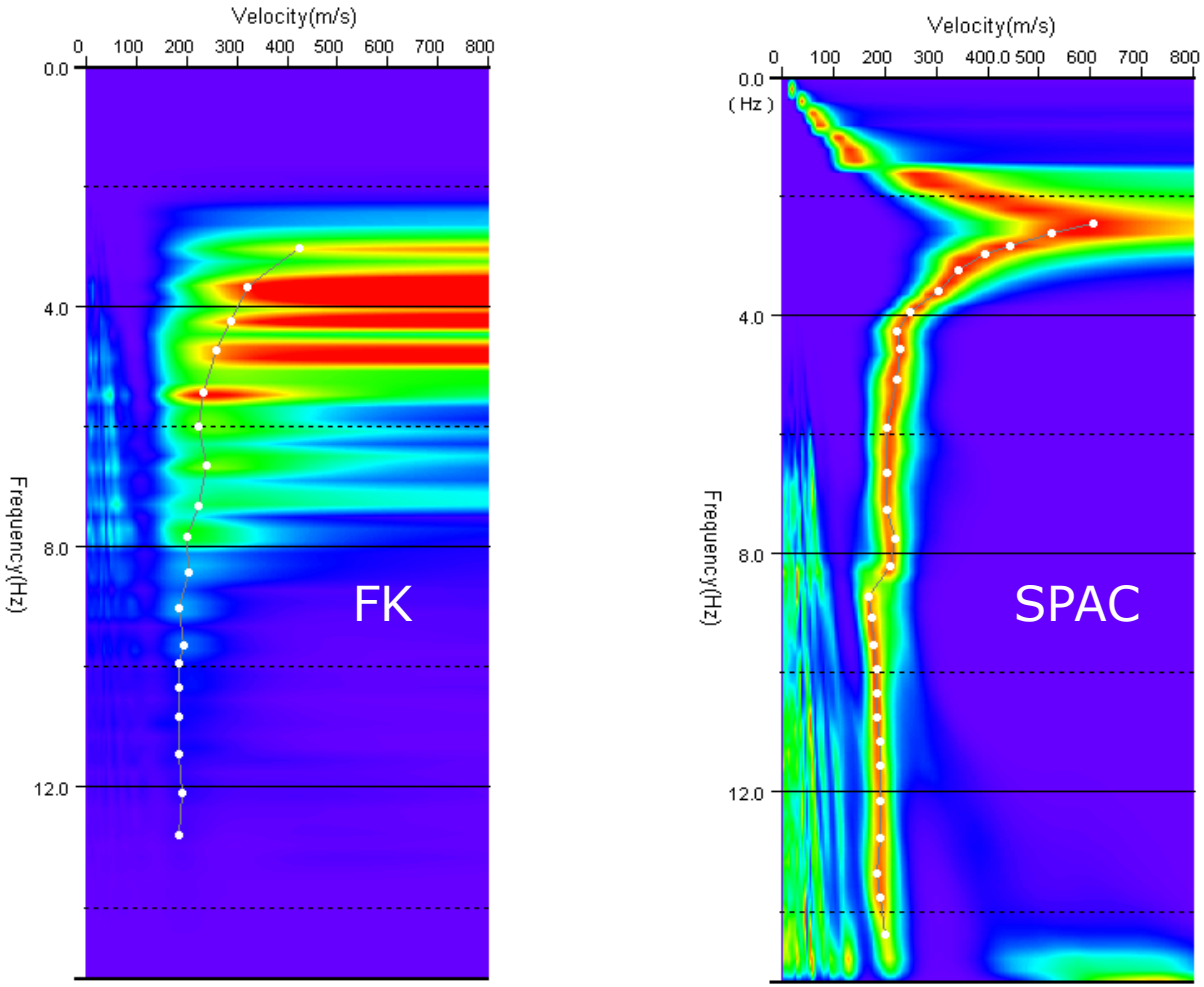
Field Example - 1



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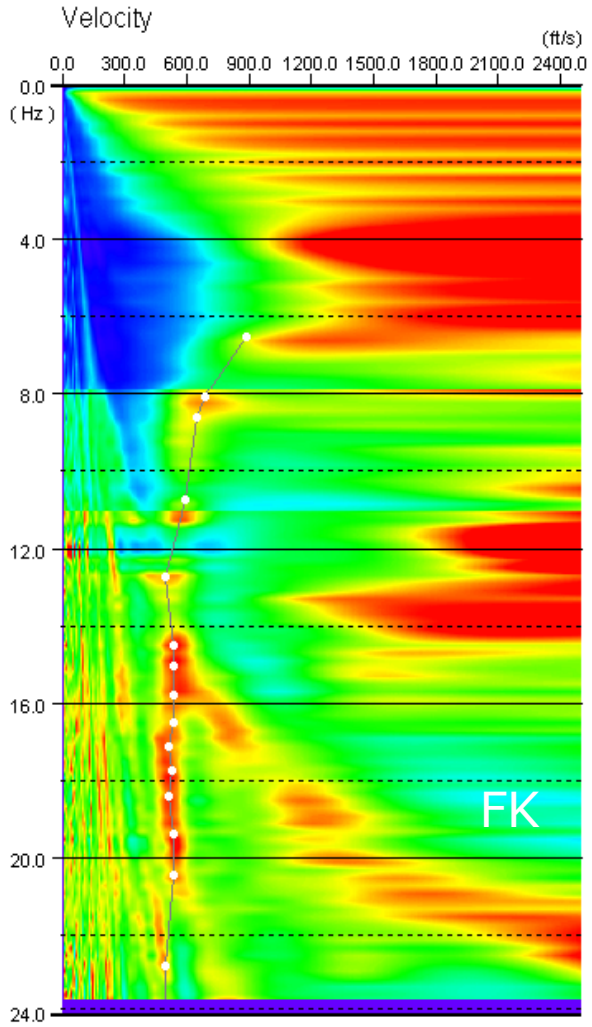
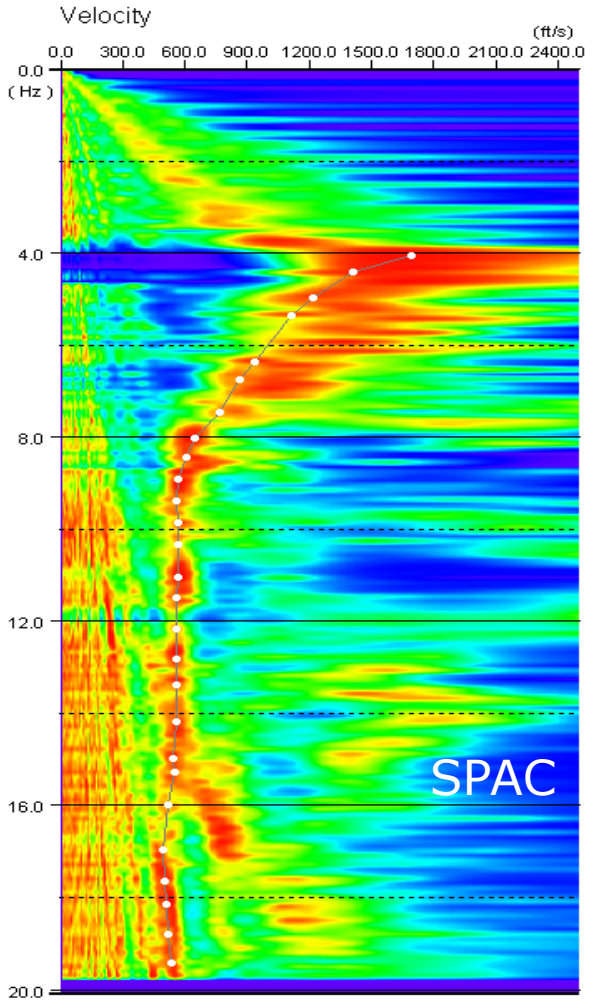
Field Example - 2





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Conclusion

- Passive surface waves carry the dispersion information
- SPAC shows higher resolution and larger frequency range for the dispersion measurement in general